SEPARATION OF BOUNDARY LAYER IN PULSATING FLOWS OF INCOMPRESSIBLE FLUID

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Any equations obtained for determining the pulsation frequency at which the boundary layer separates have a function of the relative amplitude, the pipe diameter, and the average flow rate of the fluid.

In pulsating flows, the velocity profiles differ sharply for accelerated and retarded flows. During acceleration they resemble the velocity profiles for steady flow in a gradually narrowing channel; during a period of retardation, they are closer to the velocity profiles for steady flow in an expanding channel [1, 2].

During retardation, there can be a return flow with separation of the boundary layer from the pipe wall. The research of Nikuradse [3] has shown that for water the return flow with boundary-layer separation occurs in a channel with half-angle of $\alpha \geq 5°$.

The type of pulsating flow that we shall consider has a flow-velocity variation that is very frequently encountered in practice (Fig. 1a). Within the half-period between point A and point B (Fig. 1a), the liquid flow is retarded; it is accelerated between B and C. The relative amplitude of the velocity pulsations is

$$A = \frac{w_{\max} - w_{\min}}{2w_{\text{av}}}. \tag{1}$$

The motion of the fluid between A and B can be represented as steady fluid flow in a diverging tube; at the beginning, the flow section ensures a flow rate Q for a velocity $w_{\max}$ (velocity at point A); at the outlet section, a velocity $w_{\min}$ (velocity at point B) is ensured for the same flow rate Q (Fig. 1b). The relative amplitude of the deviation between the velocity and its average value is represented by Eq. (1), where w_{av} is the average velocity that ensures the flow rate Q for a certain average value of diverging-tube flow section.

If a particle of the fluid had velocity $w_{\max}$ at point A, then following the half-period T/2, having traversed the distance L, it will have velocity $w_{\min}$ at point B. During the same time T/2, the particle can traverse the same distance L at an average velocity w_{av} in a pipe with average radius r_{av}. Thus we can write

$$L = w_{\text{av}} \frac{T}{2}. \tag{2}$$

The angle of the diverging tube that is equivalent with respect to velocity profiles to the given pulsating flow is determined as a function of $w_{\max}$, $w_{\min}$, and L. We find the angle α from Fig. 2,

$$\operatorname{tg} \alpha = \frac{r_{\max} - r_{\min}}{L}. \tag{3}$$

Substituting (2) into (3), we obtain

$$\operatorname{tg} \alpha = \frac{2(r_{\max} - r_{\min})}{T w_{\text{av}}}. \tag{4}$$

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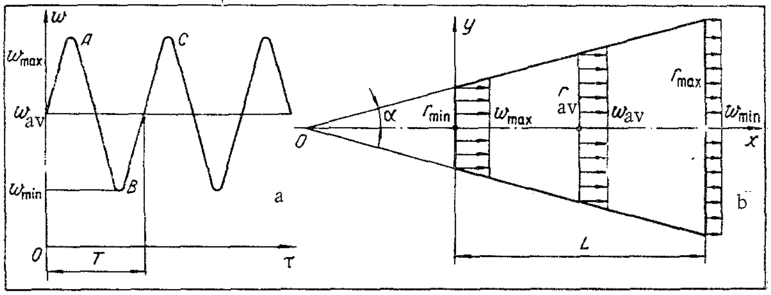

Fig. 1. Variation in flow velocity in pulsating stream (a); diagram for problem (b).

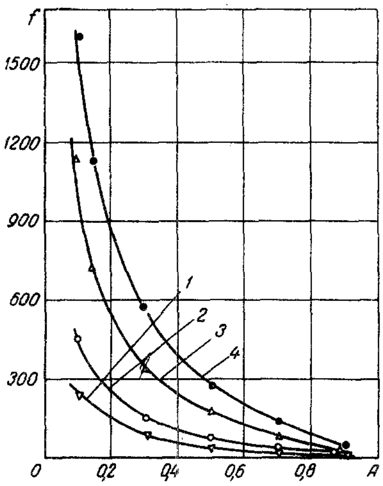

Fig. 2. Limit for boundary-layer separation, pulsating flow of water in pipe with d_{in} = 4 mm; w_{av}, m/sec: 1) 1; 2) 2; 3) 5; 4) 8.

But T = 1/f. Thus

$$\operatorname{tg}\alpha = \frac{2f(r_{max} - r_{min})}{w_{av}}. \tag{5}$$

This yields

$$\alpha = \operatorname{arctg}\frac{2f(r_{max} - r_{min})}{w_{av}}. \tag{6}$$

We change the expression for $(r_{max} - r_{min})$,

$$r_{max} - r_{min} = \frac{r^2_{max} - r^2_{min}}{r_{max} + r_{min}}. \tag{7}$$

The following equation holds for diverging and converging tubes:

$$wr^2 = \text{const.} \tag{8}$$

For small angles, we can take

$$w_{max}r^2_{min} = w_{min}r^2_{max} \cong w_{av}r^2_{av} = \text{const},$$

where

$$r_{av} = \frac{r_{max} + r_{min}}{2}.$$

This yields

$$r^2_{min} = \frac{w_{av}r^2_{av}}{w_{max}}, \tag{9}$$

$$r^2_{max} = \frac{w_{av}r^2_{av}}{w_{min}}. \tag{10}$$

Substituting (9) and (10) into (7) and simplifying, we obtain

$$r_{max} - r_{min} = \frac{w_{av}r^2_{av}}{r_{max} + r_{min}}\left(\frac{w_{max} - w_{min}}{2w_{av}}\right)\left(\frac{2w_{av}}{w_{max}w_{min}}\right)$$

or

$$r_{max} - r_{min} = \frac{w_{av}r_{av}A}{2}\left(\frac{2w_{av}}{w_{max}w_{min}}\right). \tag{11}$$

From (6) and (11), we have

$$\alpha = \operatorname{arctg}\left[fr_{av}A\left(\frac{2w_{av}}{w_{max}w_{min}}\right)\right]. \tag{12}$$

We find the function

$$Z = \frac{2w_{av}}{w_{max}w_{min}} = F(A).$$

This relationship can easily be plotted if we specify certain values of w_{av}, w_{max}, w_{min}. Analysis of the function Z = F(A) yields an equation of the form

$$Z = aA\,\mathrm{tg}\left(\frac{\pi}{2}\,A\right) + b. \tag{13}$$

For different specified values of w_{av}, w_{max}, and w_{min} we found equations for the coefficients a and b in the range of relative amplitudes $0.09 \leq A \leq 0.9$:

$$a = \frac{1.45}{w_{av}}, \tag{14}$$

$$b = \frac{2.00}{w_{av}}. \tag{15}$$

From (12), (13), (14), and (15) we obtain the final expression for the angle α:

$$\alpha = \mathrm{arctg}\left\{fr_{av}A\left[\frac{1.45}{w_{av}}\,A\,\mathrm{tg}\left(\frac{\pi}{2}\,A\right) + \frac{2.00}{w_{av}}\right]\right\}. \tag{16}$$

From (16), we find the flow-rate pulsation frequency at which separation occurs:

$$f = \frac{w_{av}\mathrm{tg}\,\alpha}{r_{av}A\left[1.45A\,\mathrm{tg}\left(\frac{\pi}{2}\,A\right) + 2.00\right]}. \tag{17}$$

Remembering that at $\alpha \geq 5°$ we have boundary-layer separation and that tan 5° = 0.09, we obtain

$$f = \frac{0.09w_{av}}{r_{av}A\left[1.45A\,\mathrm{tg}\left(\frac{\pi}{2}\,A\right) + 2.00\right]}. \tag{18}$$

It follows from (18) that the greater the relative pulsation amplitude and the larger the pipe diameter, the lower the frequency at which there will be a return flow with boundary-layer separation. Conversely, the lower the amplitude and the smaller the pipe diameter, the higher the flow-rate pulsation frequency at which separation will occur.

It also follows from (18) that as the average flow velocity increases, all other conditions being equal, the pulsation frequency at which separation occurs will go up.

Figure 2 shows curves for $f = \psi(A)$, plotted from Eq. (18) for a pipe with d_{in} = 4 mm, for average flow velocities of w_{av} = 1, 2, 5, and 8 m/sec. The curves were determined for specified boundary-layer separation boundary conditions for pulsating flows of an incompressible liquid. Equation (17) is common to many liquids, while (18) can only be used for water.

To conclude, we note that in pulsating flows the boundary layer also pulsates, and varies along the length of the pipe. Thus (18) better reflects the nature of the process for short pipes where the boundary layer is thin as compared with the channel diameter. Moreover, the nonsteady effects occurring in a pulsating boundary layer can have a substantial influence on the instant of separation. Since the process is so complex, the degree of this influence cannot be investigated theoretically, but can only be studied experimentally.

NOTATION

w_{max}, w_{min}, w_{av}	are the maximum, minimum, and time-averaged fluid flow rates in the pulsating flow, m/sec;
α	is the half-angle for a diverging tube, deg;
A	is the relative velocity-pulsation amplitude;
Q	is the time-averaged fluid flow rate;
r_{max}, r_{min}, r_{av}	are the maximum, minimum, and average radii of the diverging tube, l;
L	is the distance that a fluid particle travels during one-half a period, m;
T	is the pulsation period, sec;
f	is the pulsation frequency, Hz;

τ is the time, sec;

d_{in} is the inside diameter of the pipe, mm;

x, y are the coordinate axes.

LITERATURE CITED

1. H. Schlichting, Boundary Layer Theory, McGraw–Hill, New York (1955).
2. Bai-Shi-I, Turbulent Flow of Liquids and Gases [Russian translation], IL (1962).
3. I. Nikuradse, "Untersuchungen Uber die Stromungen des Wassers in konvergenten und divergenten Kanalen," Forschungsarbeiten des VDI, No. 289 (1929).